Real-Time Groundwater Modelling

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Contents

• What is real time modelling?
• Why is it interesting for groundwater management
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Conventional vs. Real time modelling

- **Conventional modelling**
  - Given system model, initial conditions and time-dependent inputs, system state is computed over time
  - Problem: Time varying inputs are not known in advance
  - Disadvantage: Prediction deviates from reality with time
  - Main use: Interpretation of past, parameter calibration, scenario analysis

- **Real-time modelling**
  - System state is predicted over a time interval
  - Prediction is updated with actual measured data for the next time interval (Data assimilation)
  - Advantage: Prediction does not deviate too much from reality
  - Disadvantage: No conservation law
  - Main use: Prediction and real time control
Conventional vs. Real time modelling

- Conventional modelling

- Real-time modelling
Real-time modelling in geosciences

• Historical: Groundwater model in Lignite Mining
  – Model accompanying the progress of open cast mines and obtaining new data with moving wells
  – Elements to adapt: hydraulic conductivities, recharge

• Operational weather and air quality forecasting
  – Main stochastic elements: Initial conditions, turbulence

• Prediction of hurricane trajectories

• Land-atmosphere interactions
  – Real time measurement: soil moisture from remote sensing
Why real time modelling of aquifers?

• Because real-time management is required
  – Some groundwater flow systems are very dynamic (recharge, interaction with surface water bodies, extraction and injection) and daily decisions have to be made

• Measurement devices can deliver hydraulic head and other data in real time
Why now?

• All ingredients are available at reasonable cost:
  – Long term stable sensors for the measurement of piezometric head, temperature and electric conductivity
  – Data transmission technology by wire, SMS or GPRS
  – Mature data assimilation and control techniques
Case study: Hardhof Zurich
Case study: Hardhof Zurich

Groundwater works Hardhof:
A well field in a city
The model area

- Recharge from precipitation
- Lateral inflow from hills
- Lateral inflow from hills
- River Limmat
- River Sihl
- Pumping of groundwater in Hardhof area
The motivation

- Lateral inflow from hills
- Recharge from precipitation
- Lateral inflow from hills
- River Limmat
- River Sihl
- Risk of pumping polluted water from contamination spills below the city centre
Functioning of water works Hardhof

- Pumping station
- Horizontal wells
- River bank filtration (vertical wells)
- Infiltration wells S 1-6
- Recharge basins
- Infiltration wells S 7-12
- Highway A1
- Limmat river
- To the surface reservoirs
The groundwater model

• 3D Finite Element Model for unsaturated-saturated groundwater flow (SPRING)
• Hydraulic model for Limmat and Sihl (FLORIS)
• Groundwater model requires:
  – Computation of recharge (from Meteo-data)
  – Boundary fluxes (South and north)
  – River stages along Sihl and Limmat
  – Infiltration and pumping rates
  – Fixed head at western boundary
  – Hydraulic conductivity (K)
  – Storage coefficient (Porosity)
  – Leakage coefficient of river beds (Sihl and Limmat)
• Calibration with Regularized Pilot Point Method
Calibration of the model
6 piezometers as examples
How good is the groundwater model?

- Tracer experiment 1989 for design well head protection zones
- HFB A: 33'500 m³/day, Total pumping rate HFB A,C,D: 67'000 m³/day

Rhodamine: Maximum after 10 days
Fluoresceine: Maximum after 8 days

Boundary protection zone II
Problem

**Calibration Period**

**Prediction Period without up-dating**

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**PIEZOMETER 3307**

Date

01.01.2004 01.01.2005 01.01.2006 01.01.2007

Piezometric head (m)

392.5 393 393.5 394 394.5 395 395.5 396 396.5

- Measurement
- Model
Way out: Data assimilation with the Kalman filter

Model: \[ \tilde{h}_k = A\tilde{h}_{k-1} + B\tilde{q}_{k-1} + \tilde{w}_{k-1} \]

Measurement: \[ \bar{z}_k = H\tilde{h}_k + \tilde{v}_k \]

with noises \( w \) and \( v \) having mean zero and covariances \( Q \) and \( R \) respectively.

The method proceeds in two steps

**Step 1: Propagation**
The groundwater model computes the expected head distribution \( \langle \tilde{h}_k \rangle^- \) for time \( t(k) \) from the head distribution at time \( t(k-1) \) and driving forces at time \( t(k-1) \) as well as the error covariance \( P_{k^-} \) of the expected head distribution.

\[
\begin{align*}
\langle \tilde{h}_k \rangle^- &= A\langle \tilde{h}_{k-1} \rangle + B\tilde{q}_{k-1} \\
P_{k^-} &= AP_{k-1}A^T + Q
\end{align*}
\]

\[
\begin{align*}
P_{k^-} &= \left( \tilde{h}_k - \langle \tilde{h}_k \rangle^- \right) \otimes \left( \tilde{h}_k - \langle \tilde{h}_k \rangle^- \right) \\
P_{k-1} &= \left( \tilde{h}_{k-1} - \langle \tilde{h}_{k-1} \rangle \right) \otimes \left( \tilde{h}_{k-1} - \langle \tilde{h}_{k-1} \rangle \right)
\end{align*}
\]
Way out: Data assimilation with the Kalman filter

Step 2: Update

The results are corrected by means of the measurements at time $t(k)$

$$\langle \tilde{h}_k \rangle = \langle \bar{h}_k \rangle^- + K_k \left( \bar{z}_k - H \langle \bar{h}_k \rangle^- \right)$$

$$P_k = (1 - K_k H) P_k^-$$

$K$ Kalman gain: Optimal weighting on the basis of $P$ and $R$

$$K_k = P_k^- H^T \left( HP_k^- H^T + R \right)^{-1}$$

$$\lim_{R \to 0} K_k = H^{-1}, \quad \lim_{P_k^- \to 0} K_k = 0$$

Properties of the weighting:
Expected value of the head estimate is equal to the expected value of the true head (unbiased estimate)
The estimation algorithm minimizes the expected value of the square of the estimation error (minimal estimation error).
Why Ensemble Kalman Filter?

- Kalman filter method requires the covariance matrix $P$
- This matrix becomes infeasibly large: with number of nodes $n$ of the model it is of dimension $n \times n$
- The ensemble Kalman filter estimates the error covariance matrix by taking statistics over an ensemble of realisations (with respect to forcing)
Ensemble Kalman Filtering: steps

- Initial conditions (t(k-1)) + model + random forcing
- Ensemble of model predictions (t(k))
- Measurement of heads(t(k))
- Optimal combination of model predictions and measurements:

\[
\langle \tilde{h}_k \rangle_{\text{analysis}} = \langle \tilde{h}_k \rangle_{\text{prediction}} + K_k \left( \tilde{z}_k - H \langle \tilde{h}_k \rangle_{\text{prediction}} \right)
\]

\[
K_k = P_{k,\text{prediction}}^{-1} H^T \left( H P_{k,\text{prediction}}^{-1} H^T + R_{\text{measurement}} \right)^{-1}
\]

- Analysed state at t(k) and updated covariance matrix are obtained from ensemble
- Method is a type of Bayesian estimation:
  
a priori mean/error + measurement \implies a posteriori mean/error
Real-time modelling: Ensemble version

Update each replicate with linear Gaussian approx.

Propagate each replicate with nonlinear state eq.

from D. McLaughlin
Real-time modelling: Ensemble version

from P. Meier
One measurement point in Hardhof as an example (100 realizations)

Piezometric head (m amsl)

P = Prediction
U = Update
Effect of real time update

Mean absolute error of piezometric head with update and without update using 87 measurement points:

- No data assimilation: 29.2 cm
- EnKF update: 7.2 cm
- EnKF 1-day prediction: 14.2 cm
Checking effect of real time update on points without measurements

Mean absolute error for 43 measurement points:
- No data assimilation: 29.2 cm
- EnKF update: 8.5 cm
- EnKF 1-day prediction: 15.3 cm

Mean absolute error for 44 verification points:
- No data assimilation: 29.2 cm
- EnKF update: 14.6 cm
- EnKF 1-day prediction: 17.8 cm
1-day vs. 10-day prediction

Mean absolute error for 87 observation points:
- No data assimilation: 29.2 cm
- EnKF, 1-day prediction: 14.2 cm
- EnKF, 10-day prediction: 20.2 cm
## Synthesis of all experiments: 2005-2007

Overview error analysis

<table>
<thead>
<tr>
<th></th>
<th>AAE(h), prediction</th>
<th>AAE(h), assimilation</th>
</tr>
</thead>
<tbody>
<tr>
<td>87 Data, daily</td>
<td>14.2 cm</td>
<td>7.2 cm</td>
</tr>
<tr>
<td>43 Data, daily (Measurement points)</td>
<td>15.3 cm</td>
<td>8.5 cm</td>
</tr>
<tr>
<td>44 Data, daily (Verification points)</td>
<td>17.8 cm</td>
<td>14.6 cm</td>
</tr>
<tr>
<td>87 Data, every 10 days</td>
<td>20.2 cm</td>
<td>7.3 cm</td>
</tr>
<tr>
<td>No data assimilation</td>
<td>29.2 cm</td>
<td>29.2 cm</td>
</tr>
</tbody>
</table>
Further results

• The method also allows to calibrate in real-time hydraulic conductivities and leakage coefficients and is considerably faster than Monte-Carlo-type inverse methods (Reason: solution is not iterative).

• The method is robust against drifting of a single pressure sensor.

• In online-mode the method has proven to work well in adjusting the flow field continuously. When it did not work, it was due to new abstractions at building sites.
Optimal control online: Flow chart

- **day 1**
  - 0:00: Computation of 500 runs with data of day 1 and suggested infiltrations for day 2

- **day 2**
  - 0:00: Measurement of heads of day 2 and update of ensemble

- **day 3**
  - 12:00, 14:00, 18:00: Computation of optimal infiltrations for day 3
Optimal control

• Principle
  – Control infiltration in order to obtain a functioning hydraulic barrier with minimal infiltration rate
  – Each controller for one of seven infiltration basins or well groups is responsible for keeping the local gradient at closest gradient measurement couple positive
Control points at water works Hardhof

- Horizontal wells
- Infiltration wells S 1-6
- Infiltration wells S 8-10
- Infiltration wells S 11-12
- Recharge basins
- River bank filtration (vertical wells)
- Highway A1

Couples of measurement points →
\[ \Delta h = \text{head inside} - \text{head outside} \]
**Goal criteria**

Reference value: $\Delta h_{ref} = 0.05 \, m$

$\Sigma$ infiltrations $\rightarrow \text{min!}$

Implies adjustment of controller parameters
Fuzzy PD-controller: transfer function curve

Control variable: recharge rate \( u \)

Piezometric head difference \( \Delta h \)

Derivative \( \Delta h/\Delta t \)

15000 m³
7500 m³
0 m³
Results of optimal control
\( \Delta h \)-criterion (average over period 600 days)

**Historical operation**
Distribution of average pumping and injection rates in \( \text{m}^3/\text{d} \)

**Simulated optimal operation**
New distribution of average injection rates in \( \text{m}^3/\text{d} \)
Optimal management

Injections necessary to fulfill $\Delta h$-criterion (average over period: 600 days)

$\Sigma$ injections in m$^3$/d

- Historical situation (simulated)
- Optimal control (simulated)
New criterion under trial: Minimize percentage of abstraction in wells originating from city area

Determined by percentage of streamlines weighted with corresponding flux
Conclusions

• The real time model on the basis of EnKF allows an excellent time step wise interpolation and prediction of heads by combining model results with measurements.

• Real time control is feasible and is in the testing phase at Hardhof water works with very good results.

• Control with $\Delta h$ criterion may be conservative, but is effective. Other criteria will be tried out.

• The EnKF method may also be used as an efficient tool for calibration of uncertain aquifer parameters ($K$, leakage factor).